Honey, I Shrunk the Cube

Matteo Golfarelli
Stefano Rizzi

University of Bologna - Italy

Summary

- Motivating scenario
- The shrink approach
- A Heuristic algorithm for shrinking
- Experimental results
- Summary and future work

DW & OLAP Analysis

- OLAP is the main paradigm for querying multidimensional databases

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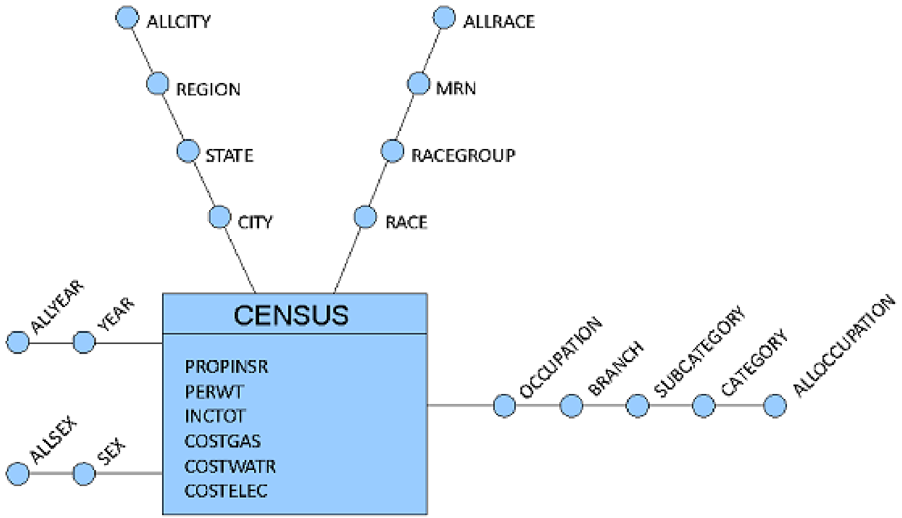

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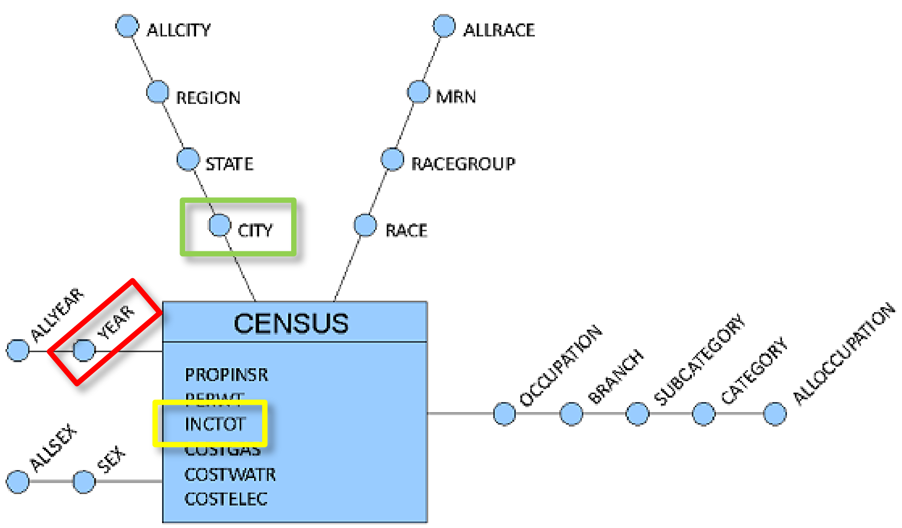
  - ✓ An OLAP query asks for returning the values of one or more numerical measures, grouped by a given set of analysis attributes



Average income in 2013 for each city

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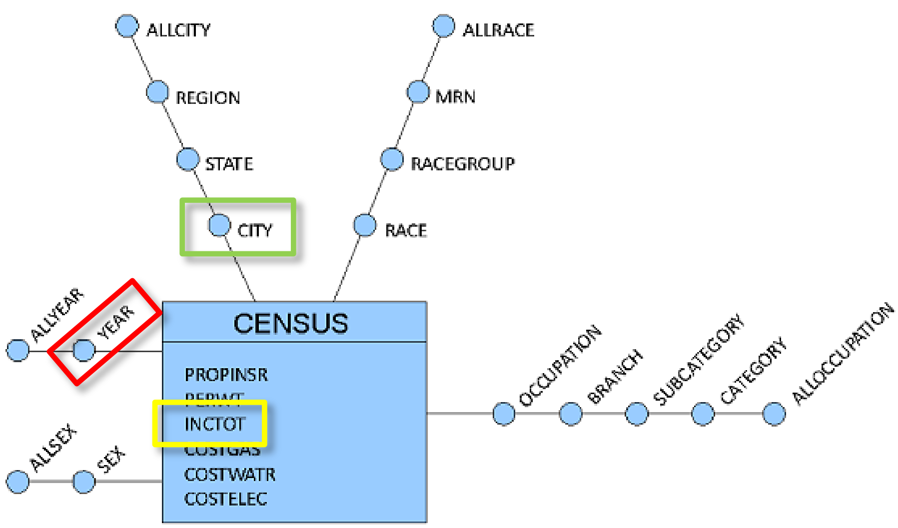

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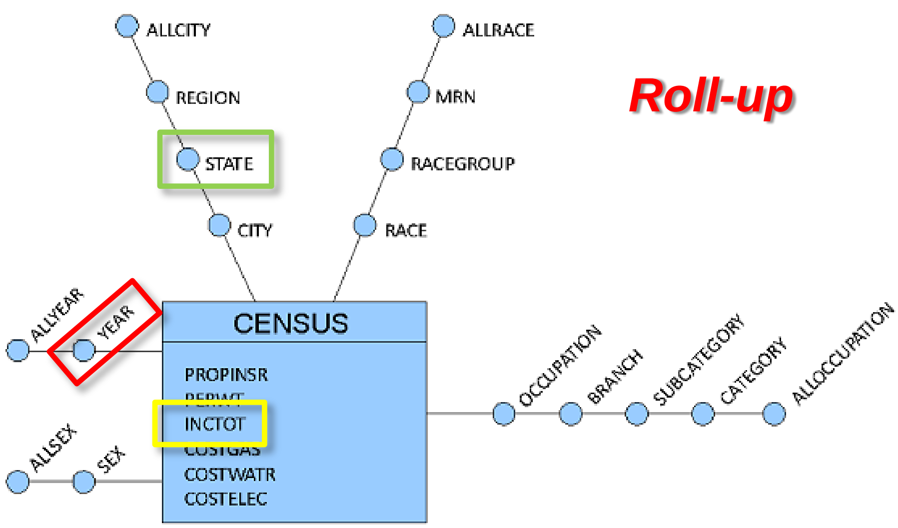


Roll-up

Average income in 2013 for each state

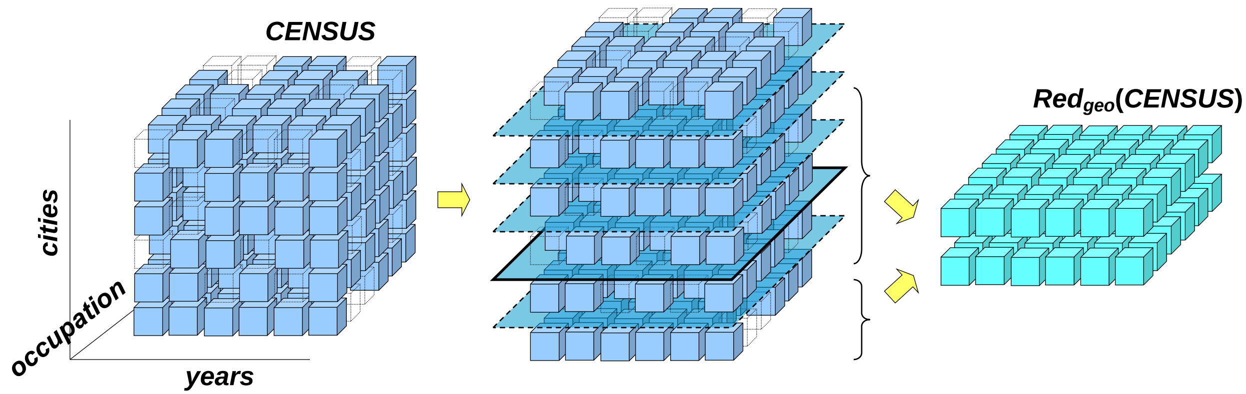
50 tuples in the resultset

Information flooding

- One of the problems that affect OLAP explorations is the risk the size of the returned data compromises their exploitation
  - ✓ more detail gives more information, but at the risk of missing the overall picture, while focusing on general trends of data may prevent users from observing specific small-scale phenomena
- Many approaches have been devised to cope with this problem:
  - ✓ Query personalization
  - ✓ Intensional query answering
  - ✓ Approximate query answering
  - ✓ OLAM On-Line Analytical Mining
- The shrink operator falls in the OLAM category
  - ✓ it is based on a clustering approach
  - ✓ **it can be applied to the cube resulting from an OLAP query to decrease its size while controlling the loss in precision**

The Shrink intuition

- The cube is seen as a set of slices, each slice corresponds to a value of the finest attribute of the shrinked hierarchy



- The slices are partitioned into a number of clusters, and all the slices in each cluster are fused into a single, approximate *f-slice* (reduction) by averaging their non-null measure values

AVG

City	2010	2011	2012
Miami	47	45	50
Orlando	44	43	52
Tampa	39	50	41
Washington	47	45	51
Richmond	43	46	49
Arlington	—	47	52

City	2010	2011	2012
Miami, Orlando	45.5	44	51
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Virginia	45	46	50.6

	2010	2011	2012
South-Atlantic	44	46	49.2

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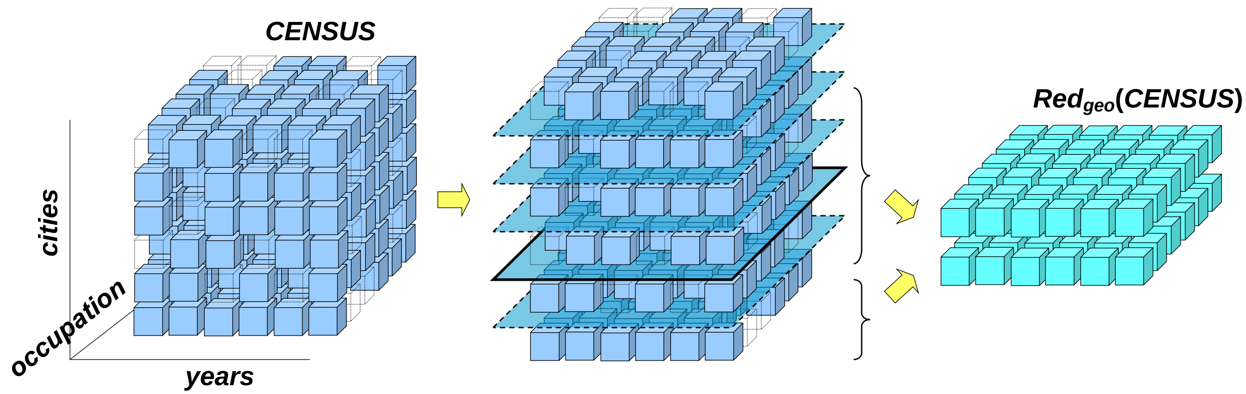

- The slices … in each cluster are fused … heir non-null measure v…

At each step the clusters to be merged must:

- **Minimize the approximation error (SSE)**
- **Respect the hierarchy structure**

Year

City	2010	2011	2012
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Year

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AVG

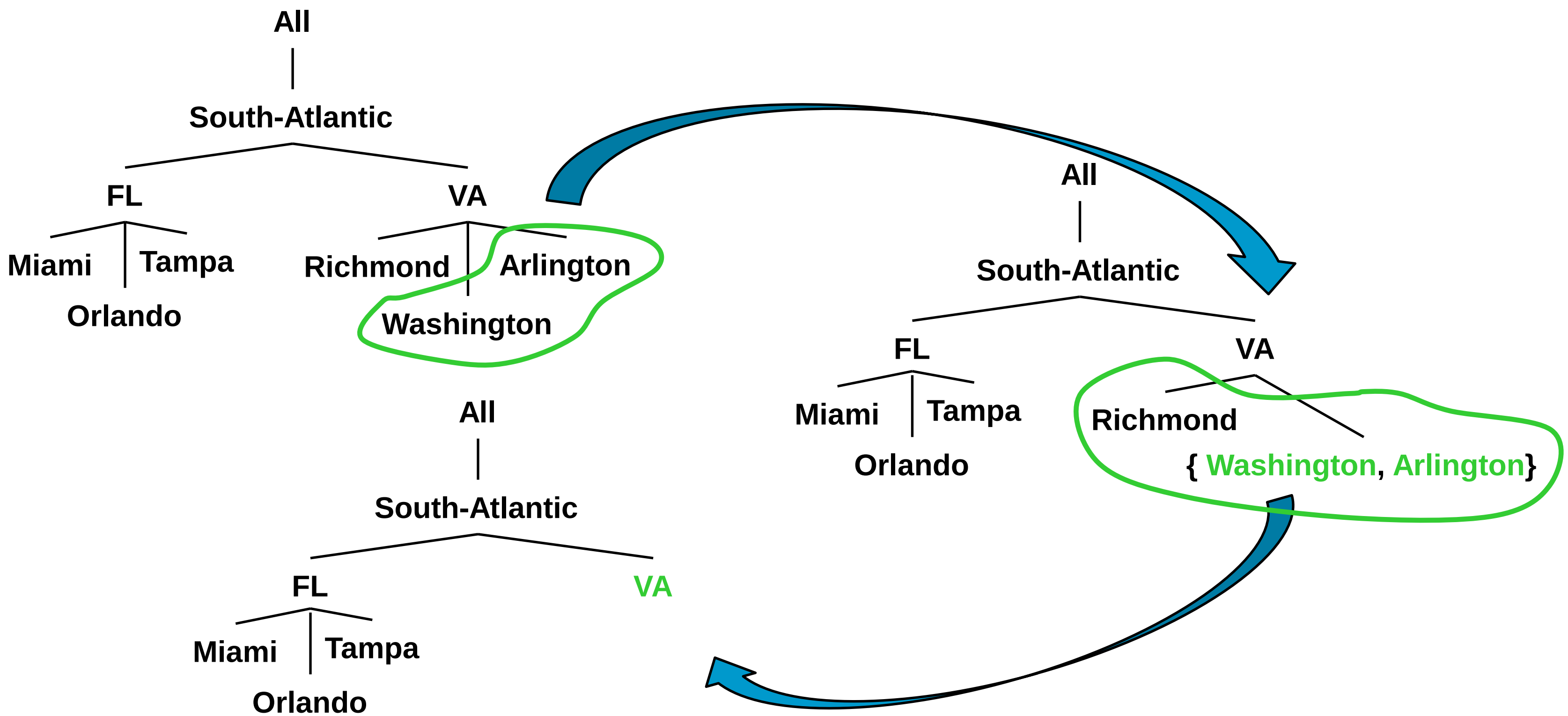
Shrink vs Roll-Up

- A roll-up operation:
  - ✓ reduces the size of the pivot table based on the hierarchy structure only
  - ✓ the level of detail is changed for all the attribute values at the same time
  - ✓ the size of the result depends on the attribute granularity and is not tuned by the user

- A shrink operation:
  - ✓ reduces the size of the pivot table considering the information carried by each slice while preserving the hierarchy structure
  - ✓ the level of detail of the result is changed only for specific attribute values
  - ✓ the size of the result is under the user control

The hierarchy constraints

- To preserve the semantics of hierarchies in the reduction, the clustering of the f-slices at each fusion step must meet some further constraints besides disjointness and completeness:
  - ✓ Two slices corresponding to values V' and V'' can be fused in a single f-slice only if both V' and V'' roll-up to the same value of the ancestor attribute



  - ✓ When a slice includes all the descendants of a given value, it is represented by that value

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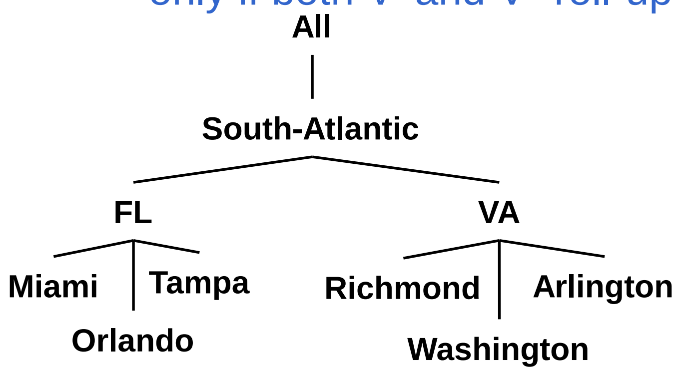

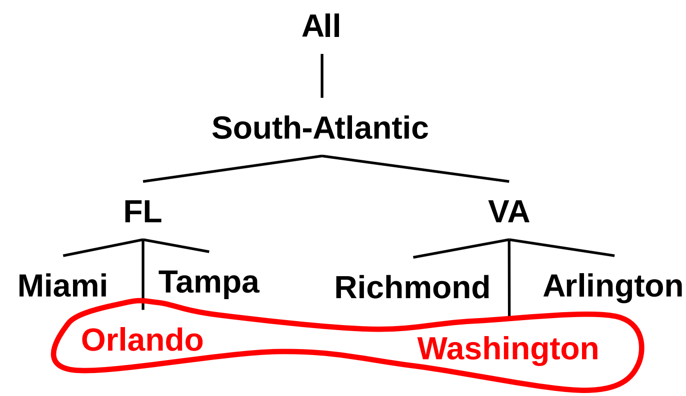

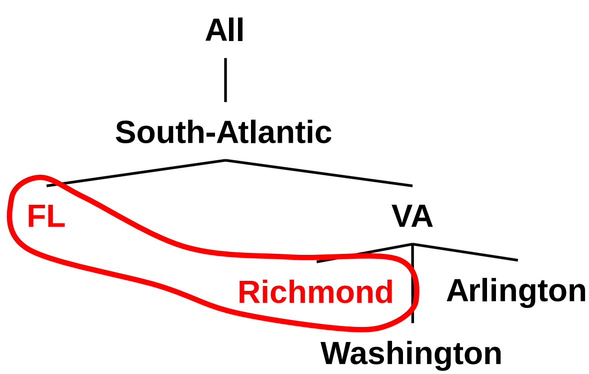

The approximation error

- The SSE of a reduction can be incrementally computed
  - ✓ The SSE of a slice V obtained merging two slices V' and V'' can be computed from the SSEs of the slices to be merged as follows:

$$SSE\left(F^{V' \cup V''}\right) = SSE\left(F^{V'}\right) + SSE\left(F^{V''}\right) + \sum_{\bar{g} \in Dom(b) \times Dom(c) \ldots} \frac{H'_{\bar{g}} \cdot H''_{\bar{g}}}{H'_{\bar{g}} + H''_{\bar{g}}} \left(F^{V'}(\bar{g}) - F^{V''}(\bar{g})\right)^2$$

  - ✓ $H'_{\bar{g}}$ is the number of non-null *V'* descendants
  - ✓ $F^{V'}(\bar{g})$ is the value of the f-slice $F^{V'}$ at coordinate $\bar{g}$

- Incremental computation of the errors deeply impacts on the computation time of the shrink algorithms proposed next

A Heuristic Algorithm

- ***Fixed size-reduction problem***: find the reduction that yields the minimum SSE among those whose size is not larger than $size_{max}$
  - ✓ The search space has exponential size
  - ✓ The presence of hierarchy-related constraints reduces the problem search space
  - ✓ Worst case when no such constraints are present: the number of different partitions of a set with |Dom(a)| elements

$$B_{|Dom(a)|} = \sum_{k=0}^{|Dom(a)|-1} \binom{|Dom(a)|-1}{k} B_k$$

⇩

A heuristic approach is needed to satisfy the real-time computation required in OLAP

A Heuristic Algorithm

- We adopted an agglomerative hierarchical clustering algorithm with constraints
  - ✓ the algorithm starts from a clustering, where each cluster corresponds to an f-slice with a single value of the hierarchy.
  - ✓ merging two clusters means merging two f-slices
  - ✓ As a merging criterion we adopted the *Ward's minimum variance method*
    - at each step we merge the pair of f-slices that leads to minimum ΔSSE increase
  - ✓ Two f-slices can be merged only if the resulting reduction preserves the hierarchy semantics
  - ✓ The agglomerative process is stopped when the next merge meets the constraint expressed by $size_{max}$

- Our approach can solve the symmetric problem too
  - ✓ ***Fixed error-reduction problem***: find the reduction that yields the minimum size among those whose SSE is not larger than SSE_{max}

A Heuristic Algorithm

City	Year: 2010	2011	2012	*SSE*
Miami	47	45	50	*0*
Orlando	44	43	52	*0*
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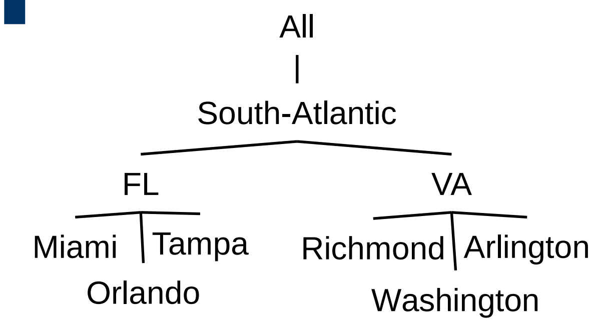
Washington	47	45	51	*0*
Richmond	43	46	49	*0*
Arlington	—	47	52	*0*
				0



A Heuristic Algorithm

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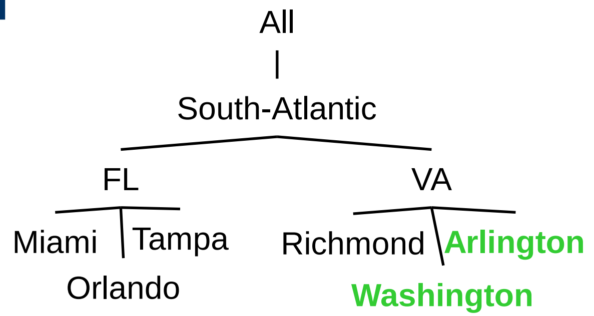
Richmond	43	46	49	*0*
Arlington	—	47	52	*0*
				0

Year

ΔSSE	Miami	Orlando	Tampa	Washington	Richmond	Arlington
Miami						
Orlando	8.5					
Tampa	85	97.5				
Washington						
Richmond				10.5		
Arlington				**2.5**	5	



A Heuristic Algorithm

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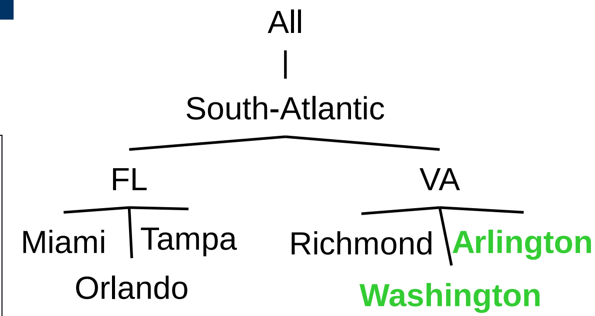

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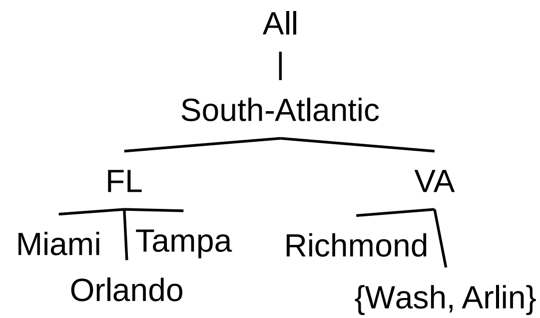

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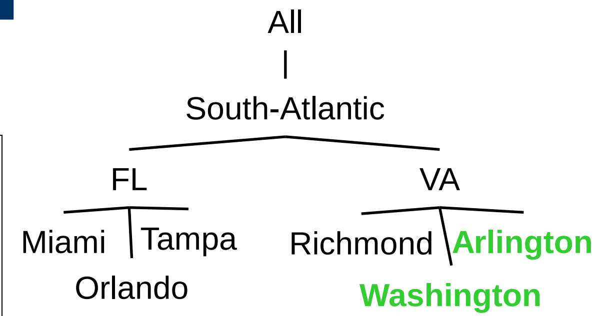

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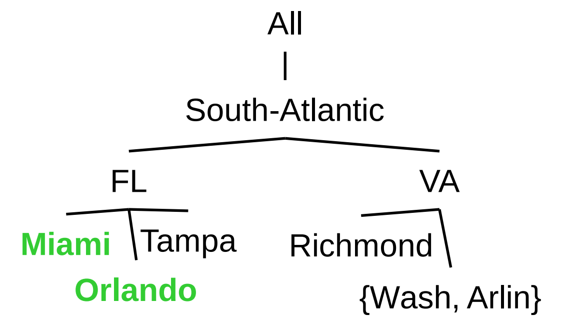

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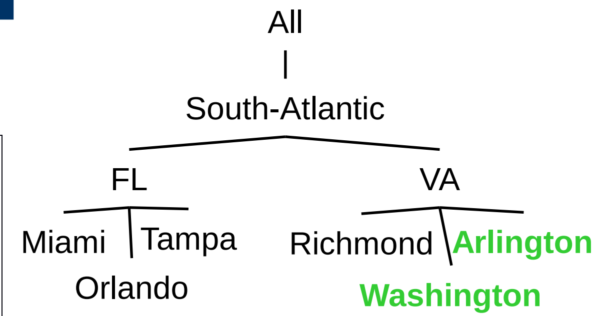

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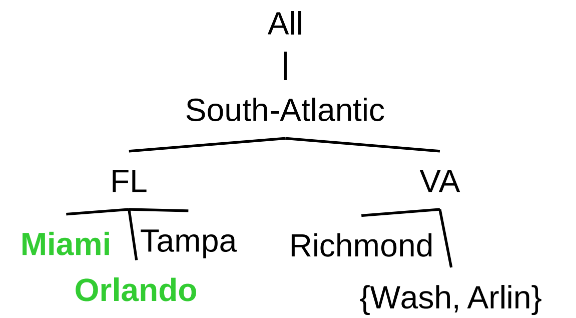

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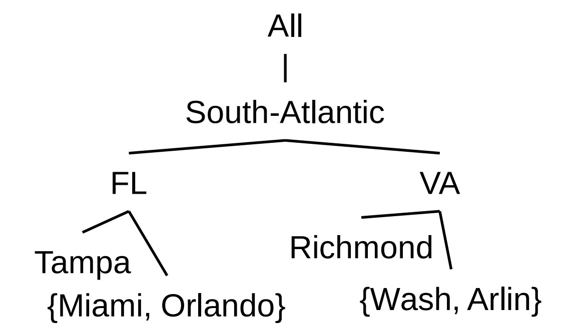

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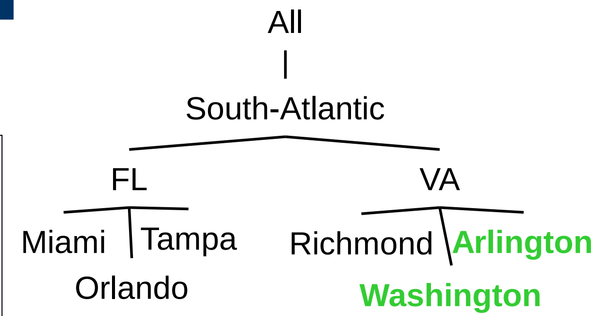

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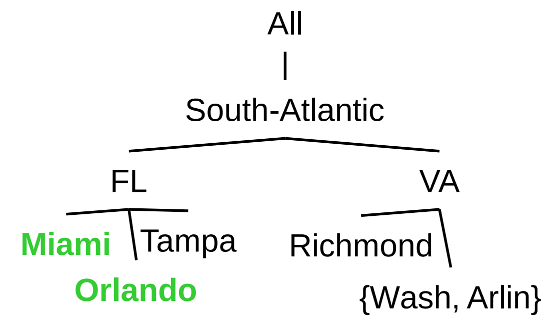

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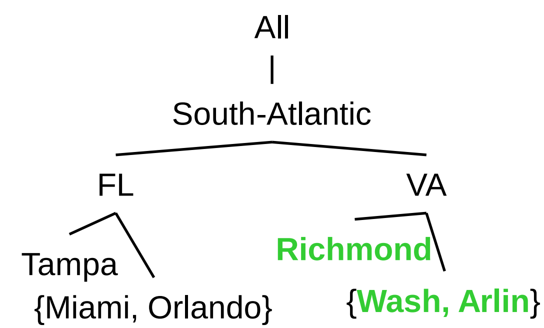
ΔSSE	Miami, Orlando	Tampa	Wash., Arlin.	Richmond
Miami, Orlando				
Tampa	127.3			
Wash., Arlin.				
Richmond			**14.7**	



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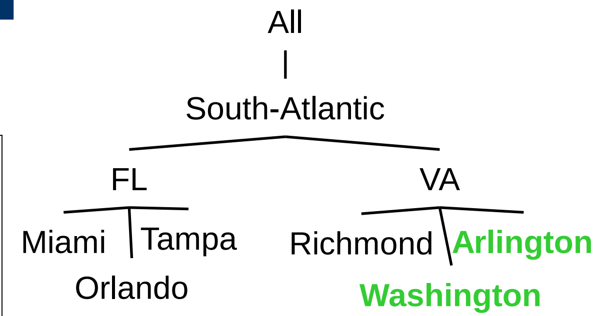

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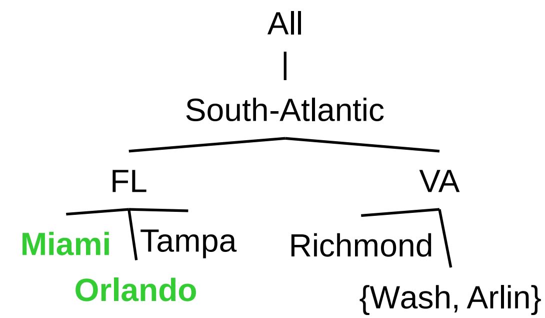

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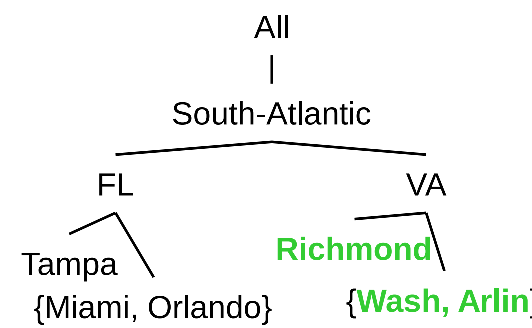

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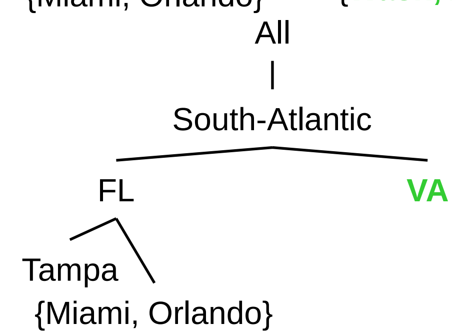
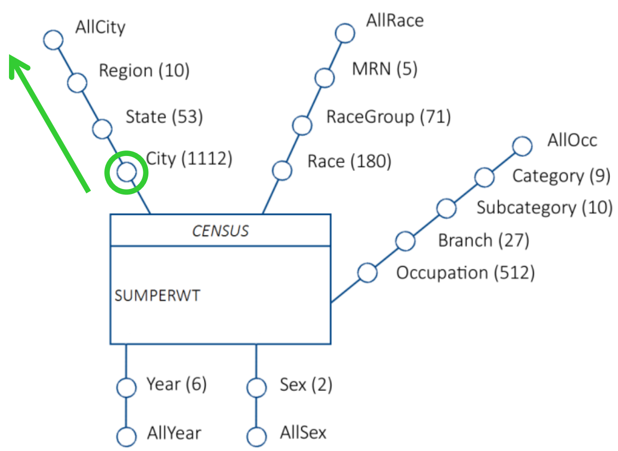
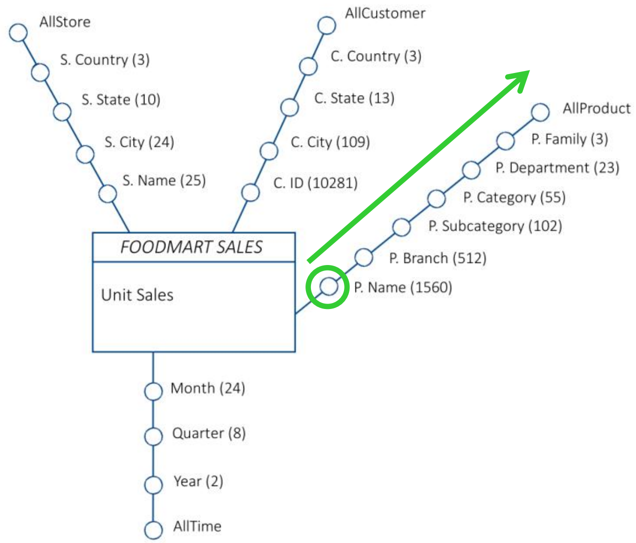


Experimental Results

- 2 different datasets adopted, 4 reduction problems
  - ✓ Different hierarchy features
  - ✓ Different sparsity
  - ✓ Different sizes

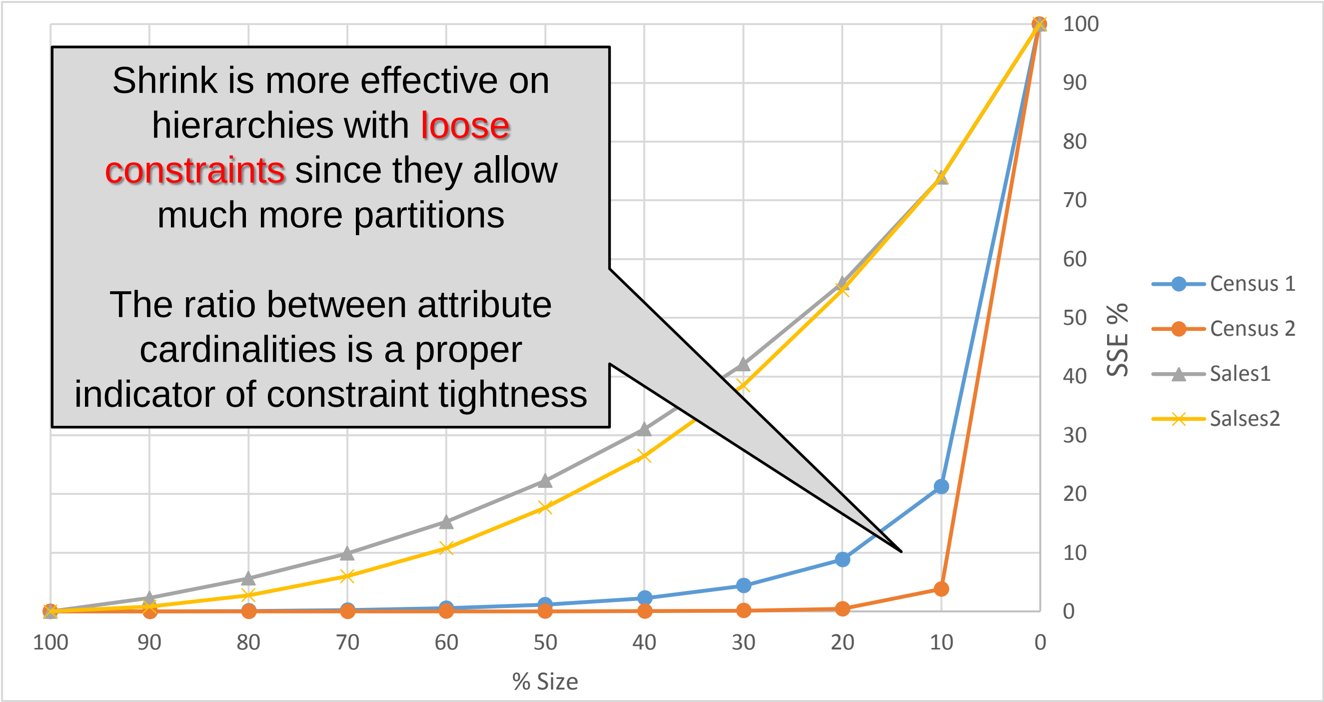




Fact	#Initial f-slice	# facts	#not-null facts	Density
Census1	1112	≈ 34 M	≈ 245 K	0,75%
Census2	1112	≈ 50 K	≈ 12 K	24,17%
Sales1	1560	≈ 34 M	≈ 200 K	0,58%
Sales2	1560	≈ 28 K	≈ 6 K	22,20%

Aprroximation errors

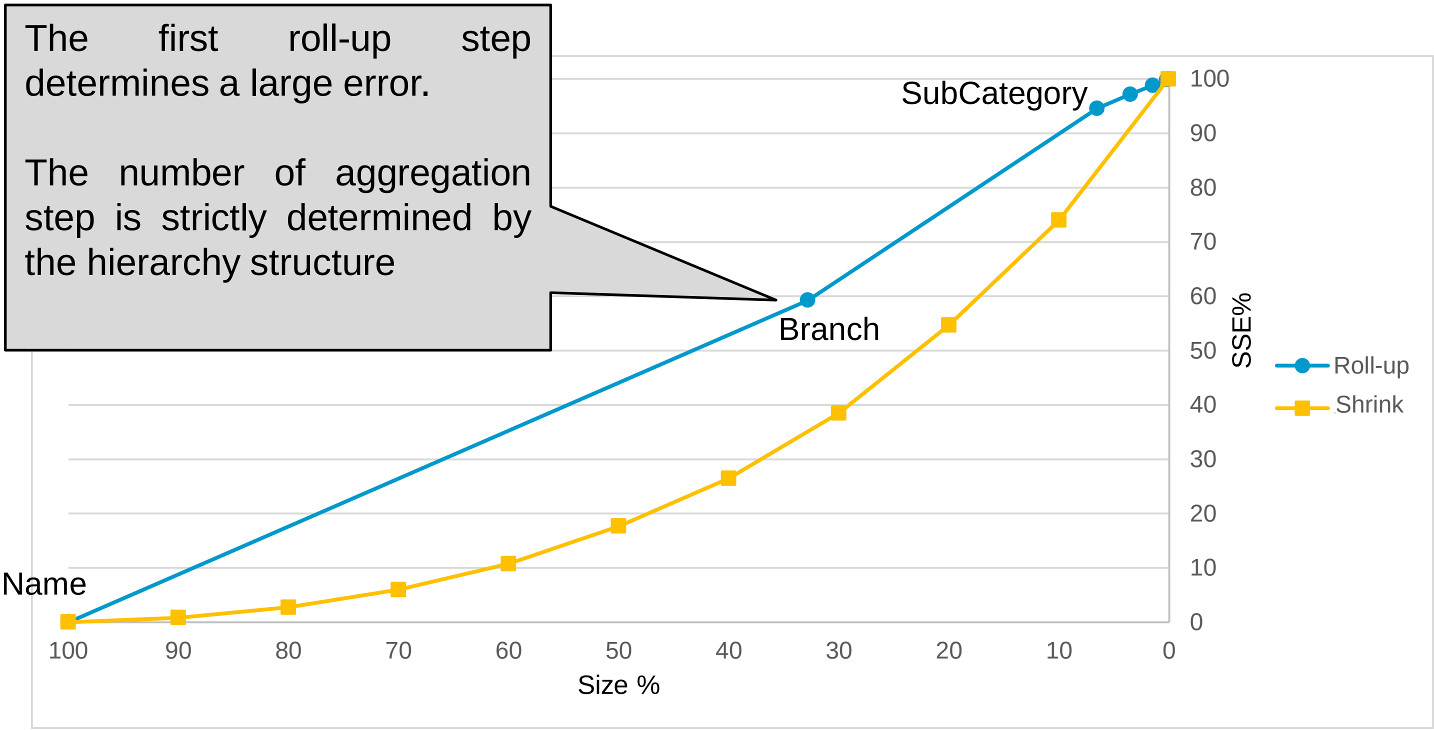
- The SSE has been normalized to allow comparisons
  - ✓ $SSE\% = \frac{SSE(Red_h(C))}{SSEMAX_h(C)}$



- Further cube features that impact on effectiveness are:
  - ✓ Sparsity: the higher the sparsity, the lower the SSE increase
  - ✓ Variance of the values: the higher the variance the cells to be merged, the higher the SSE increase

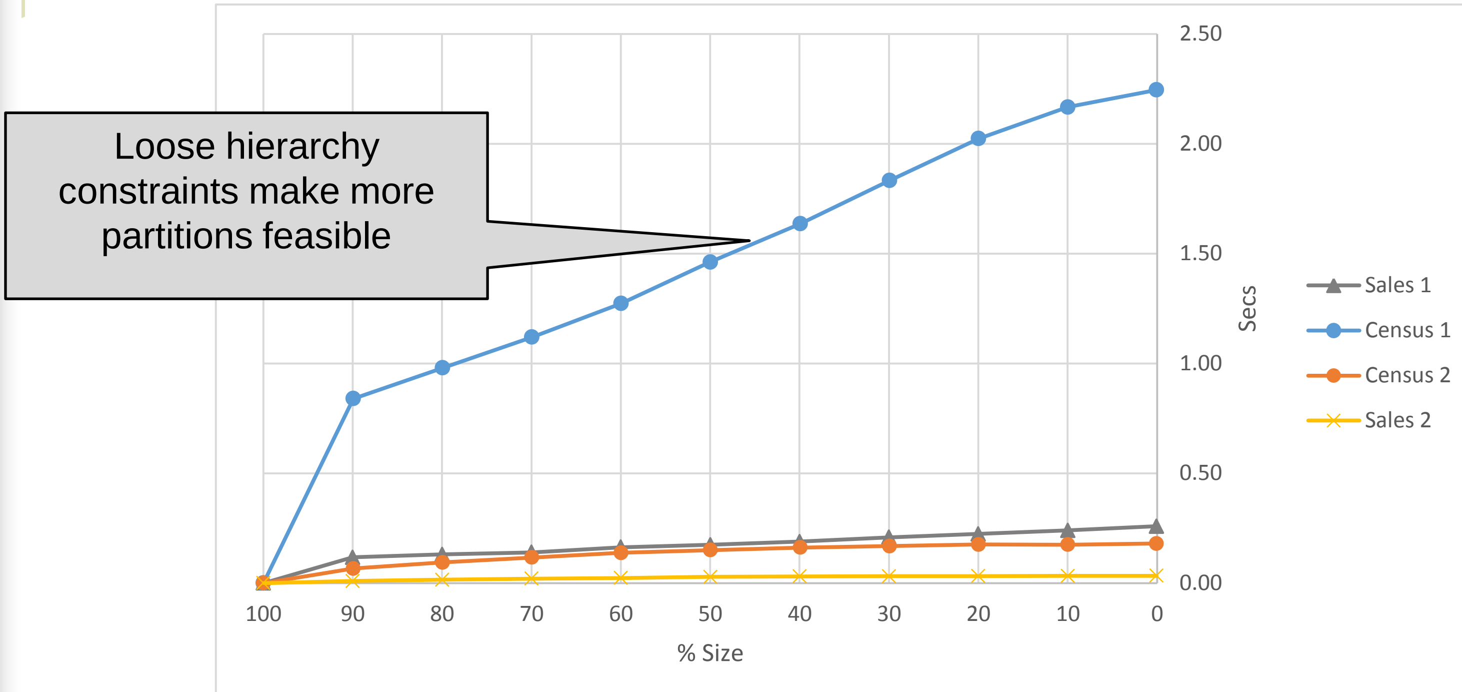
Shrink vs Roll-up

- We compared the two operators on the Sales 2 cube applying the AVG operator when rolling-up



Efficiency

- Tests are run on a Pentium i5 quad-core (2.67 GHz, 4 GB RAM)
  - ✓ Windows 7-64 bits



- Further cube features that impact on efficiency are:
  - ✓ Size of the f-slice
  - ✓ Size of the cube
- A shrink step requires less than 2 milliseconds in all of the previous test

Optimal vs Greedy

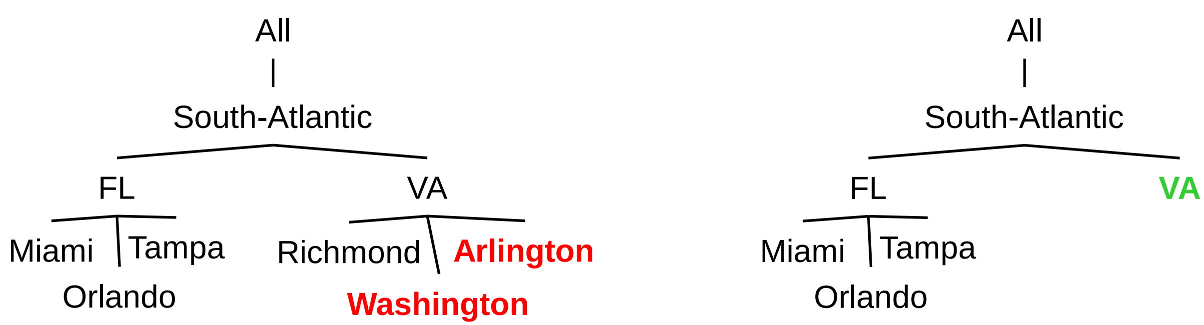
- We adopted a branch-and-bound approach based on an optimal enumeration process
  - ✓ We set $size_{max}$ = 0.3 |*Dom*(*a*)|
- Possible only on toy examples

#f-slice	# initial facts	# facts at $size_{max}$	Error	B&B execution time
23	184	90	8.31%	3 secs
24	192	90	0%	4 secs
27	135	60	0%	3 mins 12 secs
53	543	----	-----	> 6 hours

Conclusions

- Shrink: a new OLAP operation to cope with the information flooding problem
  - ✓ We proposed a heuristic implementation
  - ✓ We analyzed its effectiveness and efficiency

- Now working on:
  - ✓ Effectiveness: extending the formulation of the operator to work on several hierarchies simultaneously
  - ✓ Efficiency: studying smarter heuristics and different implementations of the shrink idea
    - The *eager* shrink operator collapses at each step all the children of a given value



  - ✓ Optimality: studying optimal algorithms exploiting a column generation technique in a set partitioning formulation
  - ✓ Visualization: find out visual metaphors for representing complex pivot tables